**Patients Queueing System for Single Doctor**

**Abstract:**

**Experience of standing in a queue and waiting for a service would not be new to an individual. The service can be of different kind in different environments. One such example of an environment (or workplace) is that of a Hospital or a Clinic where the Doctors are the servers and the Patients are the customers.**

**This study considers a single server queueing system with customers arriving for service and waiting for service if not served immediately. The exclusive aspect under study is the change in the queue and the value of the parameters like average waiting time, average queue length and Doctor’s Free Time under the cases of Relapse for a patient. The model follows the FCFS discipline and the first customer to arrive is served first.**

**Every relapse is treated as a new arrival and the arrival time after relapse is recorded as the service end time for the same customer in the service before. We generate data using Probability distribution for Arrival Time and Service Time respectively. The model estimates the queue lengths at each arrival, waiting time for each patients and also the results for Average Queue Length and Average Waiting Time.**

**Key Words: Queue, Waiting Time, Relapse, Distribution.**

1. **Introduction:**

**Queueing theory was developed to create models that predict the behaviour of systems that attempt to provide service for demands randomly arising. The first problem identified for a queueing problem was that of a phone traffic congestion. The subject has seen an aggressive expansion since then and finds applications in any industry providing any kind of service.**

**The basic characteristics of a queueing system are:**

1. **Arrival pattern of customers**
2. **Service pattern of customers**
3. **Queue discipline**
4. **System Capacity**
5. **Number of service channels**
6. **Number of service stages**

**Arrival pattern of customers is given in terms of number of arrivals per unit of time or the inter arrival time between two successive arrivals. It is also called as the mean arrival time and mean inter arrival time. Usually in a system there is uncertainty in the arrival pattern and hence the given mean inter arrival time is just a central tendency of the input process which is described by a probability distribution (Poisson or Exponential). If** λ is defined as the numbers of arrivals per unit time then 1/ λ will be the mean time between successive arrivals.

Similarly Service pattern of customers is described by number of customers served per unit time. One important condition associated with service time or service rate is that the system should not be empty. If the system is empty it implies that the service facility is not being used and is hence idle. The Service pattern can also be either probabilistic or deterministic. In this study we take the service pattern to be deterministic.

The queue discipline that is used to serve the customers on arrival can be of different types. While most of the systems follow FCFS (First Come First Served), there are a few systems that follow LCFS (Last Come First Served). In case of a Patient Queue we use the FCFS discipline for the treatment of patients.

**The queueing system can be either of finite or infinite capacity with a single or multiple servers in the system. Few service systems can have more than one stage of services as well.**

1. **Model Conceptualisation:**

**To create the queueing model, we would need the parameters arrival time, wait time, service start time, service time and service end time.**

**Arrival Time:**

**The mean arrival rate of the customers (**λ**) has been provided and the arrival of patients in a Clinic or a Hospital is expected to be random and uncertain. Hence we use the Poisson distribution to describe the Arrival Time values by using mean inter arrival time (1/** λ**). Since 1/**λ is the central tendency of the inter arrival time, a random function that generates Poisson numbers gives the Inter Arrival Time for the current patient to the next arrival. Hence the next arrival time is generated by adding the inter arrival time generated with the previous arrival time.

The Poisson distribution perfectly describes the arrival time of patients since the distribution is discrete and it gives out a limited numbers of possible observations. We explain how we use Poisson distribution to generate arrival time for patients.

Algorithm:

N = Number of Patients

aT = Arrival Time

if N = 0:

aT = random.poisson(1/ λ)

else:

aT = aT + random.poisson(1/ λ)

**In Python Random Poisson Numbers can be generated with use of function available in numpy library i.e. np.random.poisson(lam). For lam we enter the mean inter arrival time and the function draws samples from a Poisson distribution for the given** λ **value.**

**Wait Time:**

**The waiting time for the patient in the system will be zero since no one is present in the system. For the patients arriving after the first one there can be two case. Suppose for jth patient:**

1. **Service End Time (Patient[j-1]) <= Arrival Time (Patient[j]) :**

**Wait time = 0**

1. **Service End Time (Patient[j-1]) > Arrival Time (Patient[j]):**

**Wait Time = Service End Time (Patient [j-1]) - Arrival Time (Patient[j])**

**Service Start Time, Service Time and Service End Time:**

1. **Service Start Time is the sum of the Arrival Time and Wait Time.**
2. **Service Time as discussed above is deterministic and hence constant.**
3. **Service End time is the sum of Service Start Time and Service Time.**

**Relapse Cases:**

**The probability (p) is given for the case that a patient may relapse. The distribution that is used in such cases is Binomial distribution which in fact is a Bernoulli distribution (n = 1) which gives result as either a success or failure.**

**The Relapse Cases are also expected to be random and uncertain and hence we generate random Bernoulli numbers using a function from the numpy library for binomial numbers.**

**Since we need to generate Bernoulli trials we have, n = 1 and the function used in Python is:**

**np.random.binomial (1, p)**

**Updating the Queue:**

**To handle the relapse case we use First in First out discipline. Every time a patient relapses, it is treated as a new entry and the parameters are updated as follows:**

**New Arrival Time [j] = Service End Time [j]**

**Sorting is done using the following rules:**

1. **Arrival Time [k] <= New Arrival Time [j] where k > j**

**jth Patient (after relapse) and kth Patient interchange position in queue**

1. **Arrival Time [k] > New Arrival Time [j]**

**No sorting required.**

**According to the sorting and new arrival times the values of other parameters also get updated.**

1. **Data Validation:**

**We need to check if the Poisson values generated randomly using the mean inter arrival time is reliable or valid, we need to group the arrival time into groups of 100 hours, ex: 1-100, 100-200 and so on.**

**The groups are divided into groups of 100 hours because the arrival rate** λ **is given to be 20 patients per 100 hrs. Once the grouping is done we just find the Summary Statistics and see if the mean is equal to the arrival rate.**

|  |  |
| --- | --- |
| Mean | 19.95808383 |
| Standard Error | 0.105386273 |
| Median | 20 |
| Mode | 20 |
| Standard Deviation | 2.358864032 |
| Sample Variance | 5.564239521 |
| Kurtosis | 7.863407237 |
| Skewness | -0.793262462 |
| Range | 27 |
| Minimum | 1 |
| Maximum | 28 |
| Sum | 9999 |
| Count | 501 |
| Confidence Level (95.0%) | 0.207054501 |

**We can clearly see that the mean is very near to our arrival rate and hence the arrival time data is valid.**

**Now coming to the relapse cases, if the percentage of relapse1 is near 30% as given in the problem, then our data is good for use.**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Relapse 1 value** | **Count of Ralapse1** |  |  |  |
| 0 | 7043 |  | Percentage of Relapse1 | 29.56296 |
| 1 | 2956 |  |  |  |
| (blank) |  |  |  |  |
| **Grand Total** | **9999** |  |  |  |

**Clearly the percentage is very close to the given percentage of relapse.**

1. **Process of Queue Generation and Updating it:**

**The values for arrival rate, service and simulation time will be given by the user.**

**Algorithm:**

1. **Generate the arrival time for the arriving patient using the random Poisson function using the mean inter arrival time (1/** λ**) as the input to the function.**
2. **If j = 0 in the list Patients, then Service start time = Arrival Time, otherwise Service Start Time = max(Arrival Time, Patients[j-1].Service End Time)**
3. **Assign t = arrival time and service time = 1/** μ
4. **Generate the Relapse 1 to Relapse 10 values using Binomial Distribution with n=1 to draw out samples of Bernoulli trials. If Relapse i =0, then Relapse i+1 =0 for i in the range (0,10)**
5. **Append the values to a list Patients by calling the initialisation function of class Patient with the values generated in steps i to iv**
6. **Append the parameters Patient Number, Arrival Time, Wait, Service Start Time, Service Time and Service End Time from the list Patients to a new list Patients1**
7. **Call the handle relapse function to update the queue in the list Patients1 according to the success or failure of the relapse case.**
8. **Repeat steps i to vii till t<simulation time**

**While the above algorithm generates the queue and updates it, the function handle relapse mentioned in step vii of the algorithm updates the queue according to the First in First Out discipline as specified in the above section of Model Conceptualisation. After updating the queue it appends the Patients [i] to the list Patients1 in case the relapse case is true with the updated values of all the five above mentioned parameters.**

1. **Results**

**We select a random seed in our code which gives us a particular number of patients for the given simulation time. The arrival rate equals 20 patients per 100 hours, service rate is 1 patient per 3 hours and simulation time is given to be 50000.**

**The summary of results achieved after 10 relapses are:**

**Number of patients: 9999**

**Total number of services: 14181**

**Average Queue Length: 1.5358578379521894**

**Average Waiting Time: 5.843484348434844**

**Mean Time in System: 10.097909790979099**

**Utilisation: 85.0629874025195**

**Free Time for Doctor: 7470**

**Percentage Free Time for Doctor: 14.937012597480503**

**Hence for the given service rate the doctor can handle the patients and the queue does not blow up. For a service rate of above 3.526 hours the queue is expected to blow out and the treatment of all the patients would become difficult to achieve.**

**Changes:**

**Queue length - distribution**